Code No: 07A4BS04 **R07** **Set No. 1**

JAWAHARLAL NEHRU TECHNOLOGICAL UNIVERSITY HYDERABAD
II B.TECH II SEM–REGULAR/SUPPLEMENTARY EXAMINATIONS MAY – 2010
MATHEMATICS FOR AEROSPACE ENGINEERS
Aeronautical Engineering

Time: 3 hours **Max Marks: 80**

Answer any FIVE Questions
All Questions carry equal marks

⋆ ⋆ ⋆ ⋆ ⋆

1. (a) For the conformal transformation w = z^2 ,show that the circle $|z-1| = 1$ transforms the cardioid r = 2(1 + cos θ) where w = r $e^{i\theta}$ in the w- plane .
   (b) Find the condition for the transformation w = (az + b) / (cz + d) to make the circle $|w| = 1$ correspond to straight line in the z-plane. [8+8]

2. (a) Determine the analytic function f(z)=u+iv where u=$\frac{2\cos x \cosh y}{\cos 2x + \cosh 2y}$, given that f(0)=1
   (b) If u is a harmonic function, show that w=u^2 is not a harmonic function unless u is a constant. [8+8]

3. (a) For the function f(z) =$\frac{2z^3+1}{z(z+1)}$, find Taylor's series valid in a neighbourhood of z=1.
   (b) Expand f(x) = $\frac{1}{z^2-3z+2}$ in the region.
      i. $0 < |z-1| < 1$
      ii. $1 < |z| < 2$ [8+8]

4. Prove that $\int_0^a x J_n(\alpha_x) J_n(\beta x)\, dx = \begin{cases} 0 \, if & \alpha \neq \beta \\ \frac{a^2}{2} J^2_{n+1(a\ \alpha)} & if\ \alpha = \beta \end{cases}$
   Where $\alpha \, and \, \beta$ are the roots of the equation $J_n(ax) = 0$ [16]

5. (a) Prove that $\int\limits_c \frac{dz}{z-a} = 2\pi i$ where c is given by the equation $|z-a| = r$
   (b) Evaluate $\int\limits_c \frac{ze^z dz}{(z+2)^3}$ where c is $|z| = 3$ using Cauchy's integral formula
   (c) Evaluate $\int\limits_c \left(\frac{e^z}{z^3} + \frac{z^4}{(z+i)^2}\right) dz$ where c is $|z| = 2$ using Cauchy's integral theorem. [5+5+6]

6. (a) Explain summation convention in tensor analysis Write out in full, the following
      i. $a_{ij}\ x^i\ x^j$ (i, j = 1,2,3)
      ii. $g_{ij}\ dx^i\ dx^j$ (i, j = 1,2,3)
   (b) Define Christoffel symbol of first and second kind. If $(ds)^2 = (dr)^2 + r^2\ (d\theta)^2 + r^2 \sin^2\theta\ (d\varphi)^2$, then find the value of [22,1]and [1, 22] [8+8]

7. (a) Two dice are thrown together. Find the probability that
      i. the sum of numbers on their faces is 9

ii. the numbers on their faces are both odd

iii. the numbers on their faces are same.

(b) A distributor receives 20%, 15%, 35% and 30% of eggs from four poultries A,B,C,D which contains rotten eggs of 1%, 2%, 2% and 1% in the supplies from A,B,C,D respectively. A randomly chosen egg was found to be rotten. What is the probability that such egg came from the poultry C? [8+8]

8. (a) A continuous random variable X has the p.d.f given by f(x) = K e $^{-b(x-a)}$ for $a \leq x \leq \infty$
where a, b, K are constants. Find K, mean and standard deviation in terms of a and b.

(b) If X is a Poisson random variable such that P(X = 1) = 3 /10, P(X = 2) = $\frac{1}{5}$
Find P(X = 0) and P(X = 3). [8+8]

⋆ ⋆ ⋆ ⋆ ⋆